

Investigation

3

Rules and Equations

In the last investigation, you used tables and graphs of relationships to find values of one variable for given values of the other variable. In some cases, you could only estimate or predict a value.

For some relationships, you can write an equation, or formula, to show how the variables are related. Using an equation is often the most accurate way to find values of a variable.

In this investigation, you will use the patterns in tables to help you write equations for relationships. You will then use your equations to compute values of the dependent variable for specific values of the independent variable.

3.1 Writing Equations

On the last day of the Ocean Bike Tour, the riders will be near Wild World Amusement Park. Liz and Malcolm want to plan a stop there. They consider several variables that affect their costs and the time they can spend at Wild World.

Getting Ready for Problem 3.1

- What variables do you think are involved in planning for the amusement-park trip?
- How are those variables related to each other?



Malcolm finds out that it costs \$21 per person to visit Wild World. Liz suggests they make a table or graph relating admission price to the number of people. However, Malcolm says there is a simple **rule** for calculating the cost:

The *cost* in dollars is equal to 21 times the *number of people*.

He writes the rule as an **equation**:

$$\text{cost} = 21 \times \text{number of people}$$

Liz shortens Malcolm’s equation by using single letters to stand for the variables. She uses c to stand for the cost and n to stand for the number of people:

$$c = 21 \times n$$

When you multiply a number by a letter variable, you can leave out the multiplication sign. So, $21n$ means $21 \times n$. You can shorten the equation even more:

$$c = 21n$$

The equation $c = 21n$ involves one calculation. You multiply the number of customers n by the cost per customer \$21. Many common equations involve one calculation.

Problem 3.1 Equations With One Operation

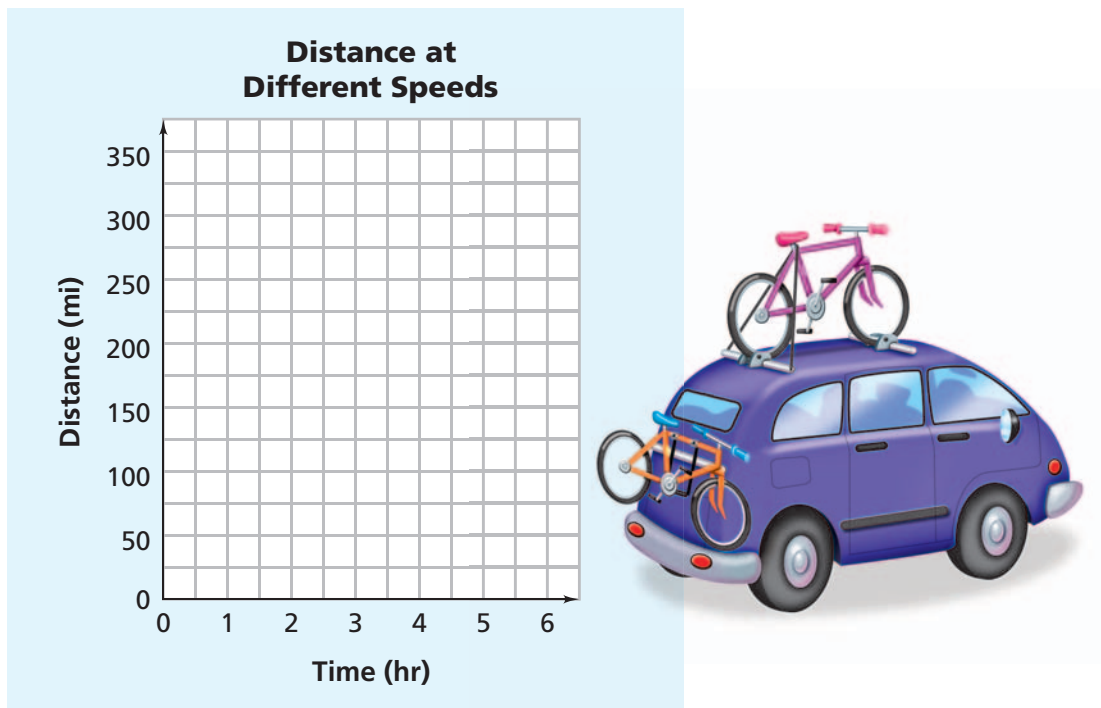
The riders visited Wild World and the tour is over. They put their bikes and gear into vans and head back to Atlantic City, 320 miles away. On their way back, they try to calculate how long the drive home will take. They use a table and a graph to estimate their travel time for different average speeds.

A. Copy and complete the table.

Distance Traveled at Different Average Speeds

Time (hr)	Distance for Speed of 50 mi/h	Distance for Speed of 55 mi/h	Distance for Speed of 60 mi/h
0	0	■	■
1	50	■	■
2	100	■	■
3	■	■	■
4	■	■	■
5	■	■	■
6	■	■	■

- B.** Copy and complete the graph for all three speeds below. Use a different color for each speed.



- C.** Do the following for each of the three average speeds:
1. Look for patterns relating distance and time in the table and graph. Write a rule in words for calculating the distance traveled in any given time.
 2. Write an equation for your rule, using letters to represent the variables.
 3. Describe how the pattern of change shows up in the table, graph, and equation.
- D.** For each speed, (50, 55, and 60 mph) tell how far you would travel in the given time. Explain how you can find each answer by using the table, the graph, and the equation.
1. 3 hours
 2. $4\frac{1}{2}$ hours
 3. $5\frac{1}{4}$ hours
- E.** For each speed, find how much time it will take the students to reach these cities on their route:
1. Atlantic City, New Jersey, about 320 miles from Norfolk
 2. Baltimore, Maryland, about $\frac{3}{4}$ of the way from Norfolk to Atlantic City

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3.2

Writing More Equations

The equations you wrote in Problem 3.1 involved only multiplication. Some equations involve two or more arithmetic operations ($+$, $-$, \times , \div). To write such equations, you can reason just as you do when you write one-operation equations:

Determine what the variables are.

Work out some specific numeric examples and examine them carefully.

What patterns do you see? What is the role of each variable in the calculation?

Write a rule in words to describe the general pattern in the calculations.

Convert your rule to an equation with letter variables and symbols.

Think about whether your equation makes sense. Test it for a few values to see if it works.

Problem 3.2 Equations With Two Operations

When Liz tells Theo about the idea to visit Wild World, he suggests she check to see whether the park offers special prices for large groups. She finds this information on the park's Web site:



- A. 1. Find the price of admission for a group of 20 people, a group of 35 people, and a group of 42 people.
2. Describe in words how you can calculate the admission price for a group with any number of people.

3. Write an equation for the admission price p for a group of n people.
 4. Sketch a graph to show the admission price for a group of any size.
 5. How does the pattern of change show up in the equation and graph? How is this pattern similar to the pattern in Problem 3.1? How is it different?
- B.** Admission to Wild World includes a bonus card with 100 points that can be spent on rides. Rides cost 6 points each.
1. Copy and complete the table below to show a customer's bonus card balance after each ride. Pay close attention to the values in the Number of Rides row.

Bonus Card Balance

Number of Rides	0	1	2	3	5	7	10	13	16
Points on Card	100	■	■	■	■	■	■	■	■

2. Describe in words how you can calculate the number of points left after any number of rides.
 3. Write an equation showing the relation between the number of rides and the points left on the bonus card. Use letters to represent the variables.
 4. Sketch a graph of the data.
 5. How does the pattern of change between the variables show up in the equation and graph? How is this pattern similar to the pattern in Question A? How is it different?
- C.** Liz wonders whether they should rent a golf cart to carry the riders' backpacks at the park. The equation $c = 20 + 5h$ shows the cost c in dollars of renting a cart for h hours:
1. Explain what information the numbers and variables in the equation represent.
 2. Use the equation to make a table for the cost of renting a cart for 1, 2, 3, 4, 5, and 6 hours.
 3. Make a graph of the data.
 4. Describe how the pattern of change between the two variables shows up in the table, graph, and equation.

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3.3

Paying Bills and Counting Profits

The students think that \$350 is a fair price to charge for the tour. Sidney wants to be certain Ocean Bike Tours will make a profit if they charge \$350. She starts making the table below.

Tour Revenue and Expenses

Number of Customers	Revenue	Bike Rental	Food and Camp Costs	Total Expenses	Profit
1	\$350	\$30	\$ 125		
2	\$700	\$60	\$250		
3	\$1,050	\$90	\$ 375		

Problem

3.3

Equations for Revenue, Expenses, and Profit

- A. Extend and complete Sidney's table for 1 to 6 customers.
- B. Write a rule in words and an equation for calculating the
 1. revenue r for n customers
 2. total expenses e for n customers
 3. profit p for n customers
- C. Use the equations you wrote in Question B to find the revenue, expenses, and profit for 20 customers and for 31 customers.
- D. Sidney forgot that the tour operators need to rent a van to carry equipment. The rental cost for the van will be \$700.
 1. How does this expense affect the equation for total expenses?
 2. How does this expense affect the equation for profit?



Homework starts on page 55.